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AN OCEANOGRAPHIC SLIDE RULE FOR COMPUTING TEMPERATURE, DEPTH, AND SALINITY

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Abstract--To simplify the processing of oceanographic data obtained at sea, a slide rule has been designed for rapid and accurate computation of temperature and depth from reversing thermometer readings, depth from wire length and wire angle, and salinity from values of temperature and electrical conductivity. The rule may be used to correct both protected and unprotected reversing thermometers, and has the advantage that individual thermometer characteristics, such as V_o and Q values and index corrections, may be recorded directly on the rule, thus eliminating the need to carry other records. Depth from thermometer readings is obtained both when Q, the pressure coefficient of the unprotected thermometer, is constant with depth, and in a special case of variable Q, hence a good estimate of depth may be made for any behavior of Q with depth. The rule also carries depth and temperature conversion scales, and has a logarithmic scale which may be used in the manner of any slide rule.

Introduction--The slide rule is designed to facilitate computation of the three basic quantities of physical oceanography, temperature, depth, and salinity. Temperature is obtained by applying corrections, found on the rule, to the reading of a protected reversing thermometer. Depth may be obtained either from the difference in the corrected readings of a protected and an unprotected reversing thermometer, or from length of wire out and wire angle at the ship. The former method is normally used for depths greater than 100 m, and the latter for shallow casts of less than 100 m [See LA FOND, 1951, for methods used heretofore]. Salinity, usually determined by chemical titration of a water sample, may also be obtained from values of the temperature and electrical conductivity of the water. The slide rule has a salinity scale based upon this relationship.

The slide rule is circular, 8-3/8 inches across, having two arms on one side and one arm on the other. The two faces are shown in Figures 1 and 2. Of the two arms shown in Figure 1, the short arm moves independently, but motion of the long arm moves both arms in such a way as to maintain a constant angle between them.

Symbols and definitions

C	=	correction for thermal expansion of a protected thermometer system subsequent to reversal
V_o	=	a constant of the individual thermometer, representing in degrees the volume of mercury below the 0° mark when the thermometer is in the reversed position at 0° Centigrade
T'	=	reading of the reversing thermometer, either protected or unprotected
t	=	reading of the auxiliary thermometer which accompanies each reversing thermometer, (that is, temperature at which reversing thermometer is read)
K	=	reciprocal coefficient of thermal expansion of the glass of which the thermometer is made; K is given on the certificate which accompanies each thermometer
C_u	=	correction for thermal expansion of an unprotected thermometer system subsequent to reversal
T_w	=	water temperature in situ, the corrected reading of a protected reversing thermometer
C'	=	value found as an intermediate step in the process of correcting an unprotected thermometer
D	=	depth at which the thermometers reversed
T_u	=	corrected reading of an unprotected reversing thermometer, a function of both temperature and pressure
ρ_m	=	mean density of the water column above the level of reversal
Q	=	pressure coefficient of the individual unprotected thermometer, expressed in degrees Centigrade increase in the reading per 0.1 kg/cm^2 increase in pressure
Wire length	=	length of wire from sea surface to water bottle
Wire angle	=	angle which the wire makes with the vertical at the ship

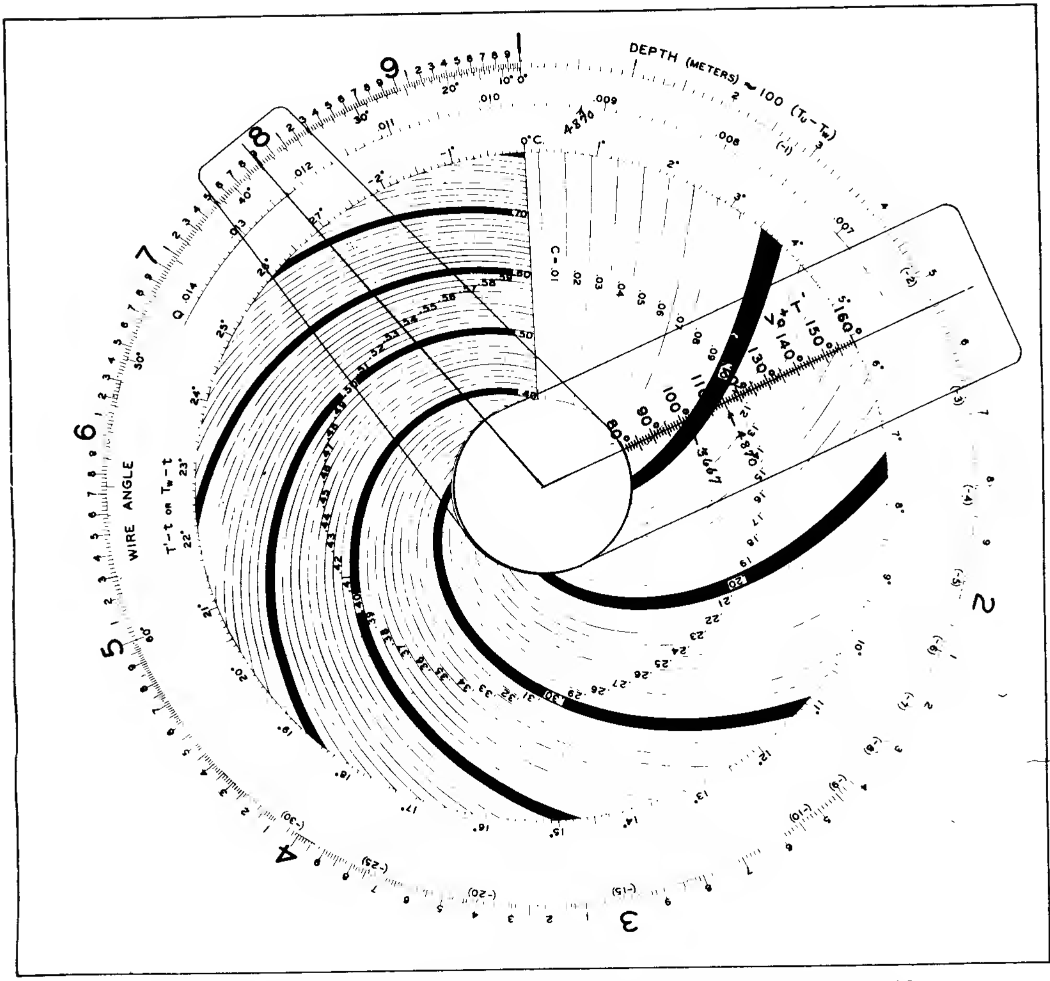

Fig. 1--Thermometer corrections and depth are determined on this side of rule; by writing thermometer numbers beside the appropriate points of the scales, as shown, the V_0 values of all thermometers may be recorded on the long arm, and the Q values of all unprotected thermometers may be recorded along the Q scale

Correction of reversing thermometer readings--The change in the reading of a protected reversing thermometer due to a change in temperature conditions subsequent to reversal may be expressed by an equation derived by SVERDRUP [1947] for protected thermometers:

$$C = (V_0 + T')(T' - t)/[K - (V_0 + T') - (T' - t)]$$

This is the correction which must be applied to the reading to obtain the reading at reversal in the sea.

The quantity K is a characteristic of the glass of which the thermometer is made, and for most thermometers has the value 6100, hence this constant value has been used in the construction of the rule. If, for either a protected or an unprotected thermometer, K has the value 6300, the thermal expansion correction C or C_u obtained from the slide rule should be reduced by about three per cent of its value. This reduction amounts to 0.01° C in the range 0.16°to 0.47°C, and 0.02°C for greater corrections.

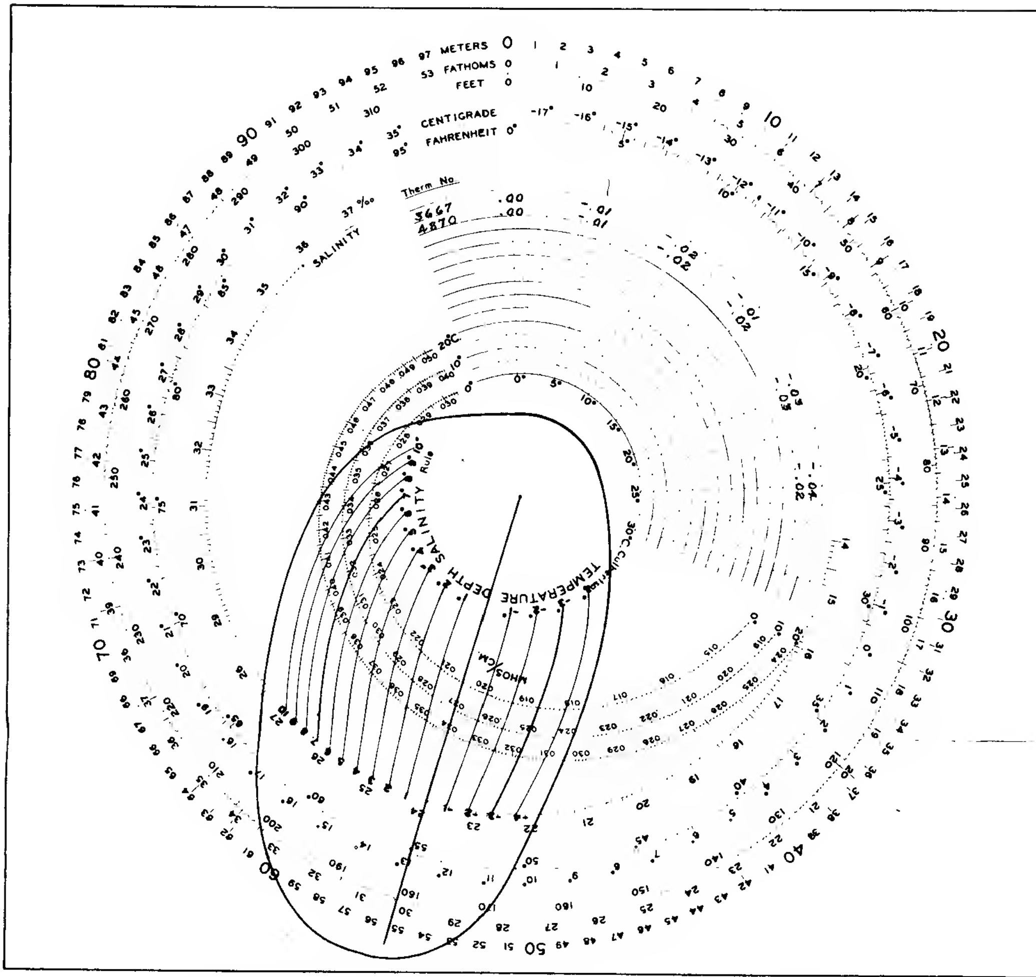

Fig. 2--Salinity is determined on this side of rule; also shown are conversion scales for depth and temperature, and a blank, semi-circular table where individual thermometer index corrections may be entered at appropriate temperatures for as many as 15 thermometers

If each expression in parentheses in the above equation be considered as a unit, the equation contains three variables, C, (V_O+T'), and $(T'-t)$. The striped graph shown in Figure 1 is a protected thermometer correction graph, giving the correction C in the body of the graph as a function of (V_O+T') on the radial scale and $(T'-t)$ on the circular scale. In plotting the stripes, negative values of C were used, for $(T'-t)$, and therefore C, are usually negative. Positive values of $(T'-t)$ are always small, and the graph is entirely satisfactory for both positive and negative values.

The rule is entered directly with the raw data T′ and t. By setting the two arms to these values on the linear scale surrounding the striped graph, the difference $(T'-t)$ is automatically measured off along this scale. If the long arm is set to the larger of the two temperatures, subsequent rotation of the long arm until the short arm is at 0° brings the scale of the long arm into position for reading the correction C beneath the value (V_O+T') on the long arm. To eliminate mental addition of V_O and T′, part of the long arm has been frosted so thermometer numbers may be entered beside their V_O values on the scale. Thus, counting T′ degrees up the scale from the V_O point, the thermal expansion correction C is obtained.

A second correction to be added to the reading of the reversing thermometer is for any error in the etching of its scale. On the side of the rule shown in Figure 2, a blank semi-circular table has been provided in which index corrections may be entered at appropriate temperatures for as many as 15 thermometers. The table is so placed that the temperature scale at its inner edge corresponds with the scale of meters at the outer edge of the rule, which is perhaps easier to read.

For unprotected thermometers the correction derived by SVERDRUP [1947] for thermal expansion subsequent to reversal is expressed by

$$C_u = (V_o + T')(T_w - t)/[K - (T_w - t)]$$

With a slight change in procedure, the protected thermometer graph of the rule may also be used to determine the expansion correction for unprotected thermometers.

From the protected thermometer equation it will be noted that the graph of the rule is the graph of an equation of the general form $C = AB/(K - A - B)$, where A is found on the radial scale, B is found on the circular scale, and C represents the value of the stripes. When dealing with unprotected thermometer data we let $(V_o + T') = A$ and $(T_w - t) = B$. Entering the graph with these variables we read from the stripes a value $C' = AB/(K - A - B)$. But the desired unprotected thermometer correction, as shown above, is $C_u = AB/(K - B)$. If on the circular scale of the rule we subtract the value C' from B, and enter the graph again with the variables A and $(B - C')$, we read from the stripes a value which is extremely close to C_u, and which may be used without error for the unprotected thermometer correction.

Let the final correction obtained in this manner be called C''; then $C'' = A(B - C')/[K - A - (B - C')]$. Since $C' = AB/(K - A - B)$

$$C'' = \frac{AB - A^2B/(K - A - B)}{K - A - B + AB/(K - A - B)} = [AB(K - A - B) - A^2B]/[(K - A - B)^2 + AB]$$

$$= AB(K - B - 2A)/[(K - B - A)^2 + AB] = [AB/(K - B)]\frac{(K - B - 2A)}{[K - B - 2A + (A^2 + AB)/(K - B)]}$$

$$= C_u \frac{(K - B - 2A)}{[K - B - 2A + (A^2 + AB)/(K - B)]} = C_u \frac{1}{[1 + (A^2 + AB)/(K^2 - 2BK - 2AK + B^2 + 2AB)]}$$

Evaluating the error in the expansion correction for unprotected thermometers due to the use of C'' instead of C_u, in the extreme case where $(V_o + T') = 160°$ and $(T_w - t) = -30°C$ we find

$$C'' = C_u[1/(1 + 20{,}800/35{,}615{,}300)] = 0.78303\ (1/1.000584) = 0.78257$$

The greatest possible difference between C'' and C_u is less than 0.0005°C. Since the correction is desired only to the nearest 0.01°C, such an error is negligible.

Along any radial line on the graph of the rule the width of the stripes is very nearly constant. This fact makes possible the extrapolation of values of the correction beyond the graph in case the value of $(V_o + T')$, for either a protected or an unprotected thermometer, is so large that it runs off the scale. The procedure is to use $1/2\,(V_o + T')$ and double the value of the correction found.

Depth determination from reversing thermometer readings--The equation used for determining depth of thermometer reversal from the corrected readings of the protected and unprotected thermometers, derived by WÜST [1933], is

$$D = (T_u - T_w)/\rho_m Q$$

The Q value of an unprotected thermometer is generally considered to be a constant. However, in determining depth on the slide rule, depth is first found assuming that the product $\rho_m Q$ is a constant. A small correction is then applied to obtain depth when Q is constant.

The scales involved in the depth determination are the outer logarithmic scale, the short Q scale, and the scale of figures in parantheses around the inner side of part of the logarithmic scale. The Q scale is a reciprocal logarithmic scale shifted to the left relative to the outer logarithmic scale by the amount 1/1.0294, where 1.0294 is an average value of ρ_m at a depth of 1000 m. Each value of Q on the Q scale is in line with the value 1/1.0294Q on the outer logarithmic scale.

In order to distinguish between them, let the symbol D represent the depth obtained when Q is a constant, and let D′ represent the depth obtained when the product $\rho_m Q$ is a constant. It is evident that D′ is directly proportional to the temperature difference $(T_u - T_w)$. Assuming that the value given for Q is correct at a depth of around 1000 m (that is, where $\rho_m = 1.0294$), the depth D′ is obtained on the outer logarithmic scale of the rule by multiplication of the factors $(T_u - T_w)$ and 1/1.0294Q, the latter found by setting to Q on the Q scale.

The difference between the depths D and D′ is effectively a function of depth alone, as shown by the computation

$$T_u - T_w = \rho_m QD = 1.0294\,QD'$$

$$D = (1.0294/\rho_m)\,D'$$

$$D - D' = (1.0294/\rho_m - 1)\,D'$$

At any depth D, the value of ρ_m is known. Therefore, the difference (D - D′) is a known function of D′. Having determined D′ on the slide rule, the depth D may be found by applying the small correction (D - D′). The figures in parentheses adjacent to the logarithmic scale are values of (D - D′) for depths greater than 1000 m, and are placed at appropriate values of D′ on the logarithmic scale. It will be noted that the values are negative, for at depths greater than 1000 m D′ is greater than D. At depths of less than 1000 meters D′ is less than D by somewhat less than one meter (see Table 1). To obtain D in shallow depths, if D′ is between 200 and 800 m, add one meter.

In order to compute the scale of (D - D′) it was necessary to assume an average distribution of ρ_m with depth. The values computed by WÜST [1933] for the North Atlantic Ocean were used. Accuracy of the values is not critical, and even in localities where the density distribution is fairly extreme, use of Wüst's values will give an error in depth rarely as great as one meter at any depth.

The first three columns of Table 1 indicate how the scale of (D - D′) was obtained. In the first column are standard values of depth, in meters. In the second column are corresponding values of ρ_m as determined by Wüst for the North Atlantic. The third column gives D′ as computed from the equation $D' = D\rho_m/1.0294$. Values of (D - D′) are found from columns 1 and 3. To find D′ at unit values of (D - D′), for placing the scale on the rule, corresponding values were plotted on a large graph, a smooth curve drawn, and the desired data read from the curve. The last two columns of Table 1 are included only for comparison, to show the relative unimportance of any particular density distribution to the computation of depth.

Table 1--D and D′ for given values of mean density ρ_m

D	Averages for North-Atlantic (Wüst)		Typical for water of low salinity	
	ρ_m	D′	ρ_m	D′
m	gm/cm^3	m	gm/cm^3	m
100	1.0262	99.7	1.0248	99.6
200	1.0267	199.5	1.0255	199.2
300	1.0271	299.3	1.0261	299.0
400	1.0275	399.3	1.0267	399.0
500	1.0278	499.2	1.0272	498.9
600	1.0282	599.3	1.0276	599.0
700	1.0285	699.4	1.0280	699.0
800	1.0288	799.5	1.0283	799.1
900	1.0291	899.7	1.0286	899.3
1000	1.0294	1000.0	1.0289	999.5
1500	1.0308	1502.0	1.0304	1501.5
2000	1.0321	2005.2	1.0318	2004.7
2500	1.0334	2509.7	1.0331	2509.0
3000	1.0346	3015.2	1.0344	3014.6
3500	1.0358	3521.8	1.0356	3521.1
4000	1.0370	4029.5	1.0369	4029.1

To facilitate the depth computation, and also to provide a convenient record of Q values, a section below the Q scale on the slide rule is frosted to permit written entries of unprotected thermometer numbers beside their Q values. It has been assumed that the value given for Q is correct at the depth where ρ_m = 1.0294. If Q is a constant, this is of course true. If Q is believed to be a variable, varying in such a way that $\rho_m Q$ remains constant, and its value has been determined at another depth, its value at ρ_m = 1.0294 for use on the slide rule may be obtained by dividing the known constant value of $\rho_m Q$ by 1.0294.

Since the slide rule gives depth both for the case where Q is a constant and for the special case where Q decreases with depth in inverse proportion to ρ_m, a good estimate of depth may be obtained for any known behavior of Q with depth. For example, if Q should increase with depth at the same rate as ρ_m, the correction to D′ would be twice as great as for Q constant.

Depth determination from wire length and wire angle--This method of depth determination is used for shallow casts, when it can be assumed that the wire is reasonably straight in the water. The slide rule scales used are the logarithmic scale and the contiguous scale of wire angles, shown in Figure 1. Each wire angle is placed at its cosine on the logarithmic scale. Depth is obtained on the logarithmic scale by multiplying wire length (below sea surface) by cosine of wire angle, the latter found by setting directly to the angle on the wire angle scale.

Salinity determination from temperature and conductivity--The salinity section of the rule, shown in Figure 2, is based upon the work of THOMAS, THOMPSON and UTTERBACK [1934], who determined the electrical conductivity of sea water as a function of its chlorinity for the six temperatures 0°, 5°, 10°, 15°, 20°, and 25°C. Since chlorinity and salinity are related, a table could be constructed from the above data which gave salinity as a function of conductivity at each of the six temperatures. Table 2 presents a small portion of this table. The salinity differences at constant conductivity through 5° temperature increases from 0°, 10° and 20° were computed and entered in the table as shown.

Table 2--Simplified portion of a table giving salinity as a function of conductivity for the six temperatures at which it was known; the salinity differences between isothermal columns have been entered as needed

Conductivity	0°	Sal. diff.	5°	10°	Sal. diff.	15°	20°	Sal. diff.	25°
0.025	29.70	4.32	25.38	22.00	2.72	19.28	17.06	1.83	15.23
0.026	31.00	4.51	26.49	22.96	2.84	20.12	17.81	1.92	15.89
0.027	32.31	4.70	27.61	23.94	2.97	20.97	18.56	2.00	16.56
0.028	33.63	4.90	28.73	24.91	3.08	21.83	19.32	2.08	17.24
0.029	34.95	5.09	29.86	25.89	3.20	22.69	20.08	2.16	17.92
0.030	36.28	5.28	31.00	26.88	3.33	23.55	20.84	2.24	18.60

The salinity portion of the rule is essentially a circular graph, where the circular coordinate is salinity, and the radial coordinate is salinity decrease per 5° temperature increase at constant conductivity (the difference columns of Table 2). The radial scale was made logarithmic in order to keep the three isotherms from crowding at one end. For better illustration, Figure 3 shows the salinity part of the rule in rectangular form.

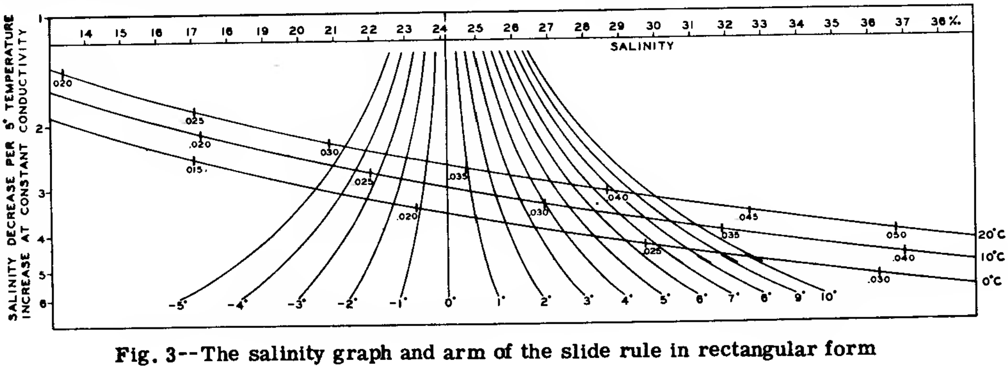

Fig. 3--The salinity graph and arm of the slide rule in rectangular form

The three isotherms were constructed by plotting points at corresponding values of salinity and salinity difference to the isotherm 5° higher, and the conductivity value was entered beside each point. It will be noted that lines of constant conductivity (not drawn) slope upward to the left through the isotherms. Every point of the graph represents a definite value of all three variables, temperature, conductivity, and salinity.

The lines of the arm represent temperature increments for use in interpolating between the isotherms of the graph, such that when a line of the arm is placed on an isotherm at a given conductivity, the straight line of the arm passes through the unmarked point of the graph which represents that conductivity and the sum of the two temperatures. The desired salinity is then read under the straight line on the scale above.

Each temperature increment line must be so placed on the arm that any point at which it crosses an isotherm is distant from the straight line by the salinity difference through the required temperature interval at the conductivity of that point of the isotherm. For example, in Figure 3 the 4° line of the arm crosses the 10° isotherm at a conductivity of 0.030 mhos/cm. The distance, measured in salinity on the salinity scale, between this point of the 4° line and the straight line is the salinity difference between 10° and 14° at a conductivity of 0.030 mhos/cm. But, as the arm moves across the graph, this distance should also be the salinity difference between 0° and 4° at a conductivity of 0.0196, and the salinity difference between 20° and 24° at a conductivity of 0.0429 mhos/cm. This is very nearly true, but not quite. It is inherent in the nature of the graph that only one line of the arm besides the straight 0° line could be made absolutely accurate with respect to all three isotherms of the rule. The selection of the ordinate scale designates the 5° line as having this property, for all three isotherms were plotted against this ordinate scale. Note that the salinity difference from any point of the 5° line to the straight line is also equal to the ordinate of the point. The other lines of the arm could be accurately placed for application to one of the isotherms of the rule, but not all three. They have been made accurate, as nearly as possible, for application to the 10° isotherm of the rule, except for the lower, off-set portions of lines 6° to 10°, which are accurate as applied to the 0° isotherm of the rule.

In order to construct the lines of the arm, it was necessary to find the salinity difference at constant conductivity between any two temperatures. This was done graphically by plotting against a temperature scale the known 5° salinity differences for each conductivity, and drawing smooth curves. The results, read from the curves, were checked for smoothness by a numerical difference method. Thus the salinity differences from each of the three isotherms 0°, 10°, and 20° to every other desired temperature were tabulated at each value of conductivity.

The arm of the rule was then plotted on a large rectangular graph similar to that of Figure 3 except that both coordinates were linear. To construct lines accurate with respect to the 10° isotherm, the salinity differences through each integral temperature increment from 10° were plotted against the salinity difference through a 5° temperature increase from 10° (the ordinate scale) at each value of conductivity. On this graph the 0° and 5° lines were of course straight. The 10° and -5° lines, which could be plotted from original data, were so nearly perfectly straight that it was believed all lines should be made straight. This provided a further check on the accuracy of the salinity differences determined graphically. Final values for plotting the arm of the rule against a logarithmic scale (see Fig. 3) were taken from this graph.

The lines of the arm thus designed for application to the 10° isotherm of the rule give accurate readings of salinity throughout the temperature range 5° to 20°C. Moreover the 0° and 5° lines of the arm are accurate as applied to all three isotherms of the rule. To determine the error involved when the other lines of the arm are applied to the 0° and 20° isotherms of the rule, the procedure outlined in the preceding paragraph was repeated using temperature increments from 0° and 20° respectively, and the corresponding lines drawn. If the lower portions of lines 6° to 10° on the arm, which do not cross the 10° isotherm of the rule, were made accurate with respect to the 0° isotherm, it was found that the error in the salinity reading could be kept within 0.02 °/oo over the entire temperature range -2° to 27°C.

Owing to construction difficulties, the present slide rule does not give the accuracy of which a salinity rule of this type is capable, as indicated above. The scales are poorest at temperatures below 5°C. Although much more accurate in most of the range, it is believed safe to say that the present rule is accurate to within 0.10 °/oo of salinity over the temperature range -2° to 27°C. The great advantage of a rule of this type is simplicity of operation, for salinity is found with but a single setting.

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